What Is Claimed Is:

1. A method for using interval techniques within a computer system to solve a multi-objective optimization problem, comprising:

receiving a representation of multiple objective functions $(f_1, \ldots, f_n)$ at the computer system, wherein $(f_1, \ldots, f_n)$ are scalar functions of a vector $\mathbf{x} = (x_1, \ldots, x_n)$;

receiving a representation of a domain of interest for the multiple objective functions;

storing the representations in a memory within the computer system; and

performing an interval optimization process to compute guaranteed bounds on a Pareto front for the objective functions $(f_1, \ldots, f_n)$, wherein for each point on the Pareto front, an improvement in one objective function cannot be made without adversely affecting at least one other objective function;

wherein performing the interval optimization process involves applying a direct-comparison technique between subdomains of the domain of interest to eliminate subdomains that are certainly dominated by other subdomains.

2. The method of claim 1, wherein performing the interval optimization process involves applying a gradient technique to eliminate subdomains that do not contain a local Pareto optimum.

3. The method of claim 2, wherein a subdomain $[\mathbf{x}]_i$ is eliminated by the gradient technique if an intersection of certainly negative gradient regions $\mathbf{C}_j$ for each objective function f_j is non-empty, $\bigcap_{j=1}^{n} \mathbf{C}_j([\mathbf{x}]_j) \neq \varnothing$;

wherein the certainly negative gradient region $\mathbf{C}_j$ for objective function f_j is the intersection of $\underline{\mathbf{N}}_j([\mathbf{x}]_i)$ (the negative gradient region associated with the minimum angle $\underline{\theta}_j$ of the gradient of f_j over the subdomain $[\mathbf{x}]_i$) and $\overline{\mathbf{N}}_j([\mathbf{x}]_i)$ (the negative gradient region associated with the maximum angle $\overline{\theta}_j$ of the gradient of f_j over the subdomain $[\mathbf{x}]_i$).

4. The method of claim 3, wherein the method further comprises iteratively:

bisecting remaining subdomains that have not been eliminated by the gradient technique; and

applying the gradient technique to eliminate bisected subdomains that do not contain a local Pareto optimum.

5. The method of claim 4, wherein bisecting a subdomain involves bisecting the subdomain in the direction that has the largest width of partial derivatives of all objective functions ($f_1, \ldots, f_n$) over the subdomain.

6. The method of claim 4, wherein the direct-comparison technique is applied once for every n iterations of the gradient technique.

7. The method of claim 6, wherein the iterations continue until either a predetermined maximum number of iterations are performed, or the largest area of any subdomain is below a predetermined value.

8. The method of claim 1,

wherein a subdomain **U** certainly dominates a subdomain **V** if every point $\mathbf{u} \in \mathbf{U}$ dominates every point $\mathbf{v} \in \mathbf{V}$; and

wherein a point **u** dominates a point **v** under minimization if,

$$u_i \leq v_i,\ i = 1, \ldots, n, \text{ and}$$

$$u_i < v_i \text{ for some } i \in \{1, \ldots, n\}.$$

9. A computer-readable storage medium storing instructions that when executed by a computer cause the computer to perform a method for using interval techniques within a computer system to solve a multi-objective optimization problem, the method comprising:

receiving a representation of multiple objective functions $(f_1, \ldots, f_n)$ at the computer system, wherein $(f_1, \ldots, f_n)$ are scalar functions of a vector $\mathbf{x} = (x_1, \ldots, x_n)$;

receiving a representation of a domain of interest for the multiple objective functions;

storing the representations in a memory within the computer system; and

performing an interval optimization process to compute guaranteed bounds on a Pareto front for the objective functions $(f_1, \ldots, f_n)$, wherein for each point on the Pareto front, an improvement in one objective function cannot be made without adversely affecting at least one other objective function;

wherein performing the interval optimization process involves applying a direct-comparison technique between subdomains of the domain of interest to eliminate subdomains that are certainly dominated by other subdomains.

10. The computer-readable storage medium of claim 9, wherein performing the interval optimization process involves applying a gradient technique to eliminate subdomains that do not contain a local Pareto optimum.

11. The computer-readable storage medium of claim 10, wherein a subdomain $[\mathbf{x}]_i$ is eliminated by the gradient technique if an intersection of certainly negative gradient regions $\mathbf{C}_j$ for each objective function f_j is non-empty,

$$\bigcap_{j=1}^{n} \mathbf{C}_j([\mathbf{x}]_j) \neq \varnothing;$$

wherein the certainly negative gradient region $\mathbf{C}_j$ for objective function f_j is the intersection of $\underline{\mathbf{N}}_j([\mathbf{x}]_i)$ (the negative gradient region associated with the minimum angle $\underline{\theta}_j$ of the gradient of f_j over the subdomain $[\mathbf{x}]_i$) and $\overline{\mathbf{N}}_j([\mathbf{x}]_i)$ (the negative gradient region associated with the maximum angle $\overline{\theta}_j$ of the gradient of f_j over the subdomain $[\mathbf{x}]_i$).

12. The computer-readable storage medium of claim 11, wherein the method further comprises iteratively:

bisecting remaining subdomains that have not been eliminated by the gradient technique; and

applying the gradient technique to eliminate bisected subdomains that do not contain a local Pareto optimum.

13. The computer-readable storage medium of claim 12, wherein bisecting a subdomain involves bisecting the subdomain in the direction that has the largest width of partial derivatives of all objective functions (f_1, ..., f_n) over the subdomain.

14. The computer-readable storage medium of claim 12, wherein the direct-comparison technique is applied once for every n iterations of the gradient technique.

15. The computer-readable storage medium of claim 14, wherein the iterations continue until either a predetermined maximum number of iterations are performed, or the largest area of any subdomain is below a predetermined value.

16. The computer-readable storage medium of claim 9,

wherein a subdomain **U** certainly dominates a subdomain **V** if every point $\mathbf{u} \in \mathbf{U}$ dominates every point $\mathbf{v} \in \mathbf{V}$; and

wherein a point **u** dominates a point **v** under minimization if,

$$u_i \leq v_i,\ i = 1, \ldots, n, \text{ and}$$

$$u_i < v_i \text{ for some } i \in \{1, \ldots, n\}.$$

17. An apparatus that uses interval techniques to solve a multi-objective optimization problem, comprising:

a receiving mechanism configured to receive a representation of multiple objective functions ($f_1, \ldots, f_n$), wherein ($f_1, \ldots, f_n$) are scalar functions of a vector $\mathbf{x} = (x_1, \ldots, x_n)$;

wherein the receiving mechanism is configured to receive a representation of a domain of interest for the multiple objective functions;

a memory configured to store the representations; and

an interval optimizer configured to performing an interval optimization process to compute guaranteed bounds on a Pareto front for the objective functions ($f_1, \ldots, f_n$), wherein for each point on the Pareto front, an

improvement in one objective function cannot be made without adversely affecting at least one other objective function;

wherein the interval optimizer is configured to apply a direct-comparison technique between subdomains of the domain of interest to eliminate subdomains that are certainly dominated by other subdomains.

18. The apparatus of claim 17, wherein the interval optimizer is configured to apply a gradient technique to eliminate subdomains that do not contain a local Pareto optimum.

19. The apparatus of claim 18, wherein a subdomain $[\mathbf{x}]_i$ is eliminated by the gradient technique if an intersection of certainly negative gradient regions $\mathbf{C}_j$ for each objective function f_j is non-empty, $\bigcap_{j=1}^{n} \mathbf{C}_j([\mathbf{x}]_j) \neq \emptyset$;

wherein the certainly negative gradient region $\mathbf{C}_j$ for objective function f_j is the intersection of $\underline{\mathbf{N}}_j([\mathbf{x}]_i)$ (the negative gradient region associated with the minimum angle $\underline{\theta}_j$ of the gradient of f_j over the subdomain $[\mathbf{x}]_i$) and $\overline{\mathbf{N}}_j([\mathbf{x}]_i)$ (the negative gradient region associated with the maximum angle $\overline{\theta}_j$ of the gradient of f_j over the subdomain $[\mathbf{x}]_i$).

20. The apparatus of claim 19, wherein the interval optimizer is configured to iteratively:

bisect remaining subdomains that have not been eliminated by the gradient technique; and to

apply the gradient technique to eliminate bisected subdomains that do not contain a local Pareto optimum.

21. The apparatus of claim 20, wherein bisecting a subdomain involves bisecting the subdomain in the direction that has the largest width of partial derivatives of all objective functions (f_1 , …, f_n) over the subdomain.

22. The apparatus of claim 20, wherein the direct-comparison technique is applied once for every *n* iterations of the gradient technique.

23. The apparatus of claim 22, wherein the iterations continue until either a predetermined maximum number of iterations are performed, or the largest area of any subdomain is below a predetermined value.

24. The apparatus of claim 17,

wherein a subdomain **U** certainly dominates a subdomain **V** if every point $\mathbf{u} \in \mathbf{U}$ dominates every point $\mathbf{v} \in \mathbf{V}$; and

wherein a point **u** dominates a point **v** under minimization if,

$$u_i \le v_i,\ i = 1, \ldots, n, \text{ and}$$

$$u_i < v_i \text{ for some } i \in \{1, \ldots, n\}.$$